MTH 2310, LINEAR ALGEBRA
DR. ADAM GRAHAM-SQUIRE, FALL 2017

MINITEST 4 REVIEW

- The test will take approximately 30-40 minutes.
- You can use a calculator on some questions, and there will likely be some questions for which a calculator is either not allowed or not of any use.
- The test will cover sections 6.1, 6.2, 6.3 and 6.5.
- To study for the test, I recommend looking over your notes and trying to rework old problems from class, HW problems, and questions from previous quizzes. You can also work out problems from the Supplementary Exercises at the end of Chapter 6. In particular # 1, 4-7, 10.

  The answers to some of those are in the back of the textbook if you want to check your work.
- As with the quizzes, it is important that you know not just the answer to a question, but also how to explain your answer.

Some problems to work on in class today (most of these are even-numbered problems from the textbook):

(1) True/False: If True, justify your answer with a brief explanation. If False, give a counterexample or a brief explanation.
  (a) $\mathbf{u} \cdot \mathbf{v} - \mathbf{v} \cdot \mathbf{u} = 0$.
  (b) The orthogonal projection of $\mathbf{y}$ onto $\mathbf{v}$ is the same as the orthogonal projection of $\mathbf{y}$ onto $c\mathbf{v}$ whenever $c \neq 0$.
  (c) The orthogonal projection of $\mathbf{y}$ onto $\mathbf{v}$ is the same as the orthogonal projection of $\mathbf{v}$ onto $c\mathbf{y}$.
  (d) If W is a subspace of $\mathbb{R}^n$ and if $\mathbf{v}$ is in both W and $W^\perp$, then $\mathbf{v}$ must be the zero vector.
  (e) The least-squares solution of $A\mathbf{x} = \mathbf{b}$ is the point in the column space of A that is closest to $\mathbf{b}$.

(2) Let $\mathbf{w} = \begin{bmatrix} 3 \\ -1 \\ -5 \end{bmatrix}$ and $\mathbf{x} = \begin{bmatrix} 6 \\ -2 \\ 3 \end{bmatrix}$. (a) Calculate $\dfrac{\mathbf{x} \cdot \mathbf{w}}{\mathbf{w} \cdot \mathbf{w}}\mathbf{w}$.
  (b) Find a unit vector orthogonal to $\mathbf{x}$ (Hint: first find an orthogonal vector by inspection and then normalize it).

(3) Suppose $\mathbf{y}$ is orthogonal to $\mathbf{u}$ and $\mathbf{v}$. Show that $\mathbf{y}$ is orthogonal to every vector in $\operatorname{Span}\{\mathbf{u}, \mathbf{v}\}$. (Hint: An arbitrary $\mathbf{w}$ in $\operatorname{Span}\{\mathbf{u}, \mathbf{v}\}$ is of the form $\mathbf{w} = c_1\mathbf{u} + c_2\mathbf{v}$.)

(4) Let $\mathbf{y} = \begin{bmatrix} 2 \\ 6 \end{bmatrix}$ and $\mathbf{u} = \begin{bmatrix} 7 \\ 1 \end{bmatrix}$. Write $\mathbf{y}$ as the sum of a vector in Span$\{\mathbf{u}\}$ and a vector orthogonal to $\mathbf{u}$.

(5) Suppose W is a subspace of $\mathbb{R}^n$ spanned by n nonzero orthogonal vectors. Explain why $W = \mathbb{R}^n$.

(6) Let W be a subspace of $\mathbb{R}^n$. Prove that dim W + dim $W^{\perp}$=n by doing the following:

(a) Suppose W has an orthogonal basis $\{w_1, w_2, \ldots, w_p\}$, and let $\{v_1, v_2, \ldots, v_q\}$ be an orthogonal basis for $W^{\perp}$. Explain why $\{w_1, w_2, \ldots, w_p, v_1, v_2, \ldots, v_q\}$ is an orthogonal set.

(b) Explain why $\{w_1, w_2, \ldots, w_p, v_1, v_2, \ldots, v_q\}$ spans $\mathbb{R}^n$.

(c) Explain why this proves that dim W + dim $W^{\perp}$=n.

(7) Find a least-squares solution for $A\mathbf{x} = \mathbf{b}$ <u>and</u> compute the least-squares error for

$$A = \begin{bmatrix} 1 & 3 \\ 1 & -1 \\ 1 & 1 \end{bmatrix} \qquad \mathbf{b} = \begin{bmatrix} 5 \\ 1 \\ 0 \end{bmatrix}$$